LUSAKA APEX MEDICAL UNIVERSITY

FACULTY OF PRE-MEDICAL SCIENCES

DEPARTMENT OF MATHEMATICS

TUTORIAL SHEET 3 ***SEP 2024 JULY –DEC INTAKE***

1. Solve the following inequalities.
   a) $8x + 2 > x + 7$
   b) $2x^2 + 7x - 15 < 0$
   c) $2(3x - 4) - (x + 6)(x - 2) \geq 0$
   d) $\left|\frac{x-3}{x-4}\right| \leq 4.$
   e) $|x| \leq |2x - 3|$
   f) $\frac{3x+5}{x-3} > 8$
   g) $|3x - 5| \geq 6$
2. Solve the following quadratic equations using factorization, completing the square and quadratic formula methods.
   a) $x^2 - 8x + 15 = 0$
   b) $x^2 - 2x - 4 = 0$
   c) $(x + 1)^2 - 8(x + 1)(x + 2) = 0$
   d) $\frac{2}{x+1} + \frac{1}{x-1} = \frac{1}{x^2+1}$
   e) $-x^2 + 6x + 18 = 0$
   f) $x^2 + 4 = 0$
3. Write the following in the form $f(x) = a(x + b)^2 + c$ where a, b, and c are constants.
   a) $f(x) = x^2 - 2mx + n$
   b) $f(x) = 2x^2 - 8x + 10$
   c) $f(x) = 4x = 3 - x^2$
   d) $f(x) = -x^2 - 5x - 10$
4. Sketch the graph of the function, showing clearly the x – intercept and y – intercepts and the turning point. State (i) the line of symmetry, and (ii) the maximum or minimum value of the function.
   a) $f(x) = 2x^2 - x - 7$
   b) $g(x) = -x^2 - 5x - 10$
   c) $h(x) = x^2 + 4$
5. Without solving the quadratic equations determine the nature of roots for each of the equations below,
   a) $x^2 + 3x - 4 = 0$
   b) $2x^2 = 12x + 54 = 0$

c) $27x^2 - 12 = 0$
d) $-x^2 + 6x + 18 = 0$
e) $x^2 - 4 = 0$

6. Determine the value(s) of p for which the quadratic equation $2x^2 + px + 8 = 0$ has
   a) real roots
   b) non real roots
7. Let α and β be the roots of the quadratic equation $x^2 + 5x + 6 = 0$. Evaluate the following.
   a) $\frac{4}{\alpha} - \frac{4}{\beta}$
   b) $\frac{\alpha}{\beta} - \frac{\beta}{\alpha}$
   c) $\alpha^3 + \beta^3$
   d) $\frac{1}{\alpha^2 - 1} + \frac{1}{\beta^2 - 1}$
   e) $\alpha^3 - \beta^3$
   f) $\frac{1}{\alpha^2} - \frac{1}{\beta^2}$
8. Let α and β be the roots of the quadratic equation $x^2 + 7x - 3$. Find the quadratic equation whose roots are
   a) $2\alpha - 1 \; and \; 2\beta - 1$
   b) $\alpha + \frac{5}{\alpha}$ and $\beta + \frac{5}{\alpha}$
   c) $\frac{2}{\alpha} and \; \frac{2}{\beta}$
   d) $\alpha^2\beta \;\; and \; \beta^2\alpha$.
9. Rahul and Rohan have 45 marbles together. After losing 5 marbles each, the product of the number of marbles they both have now is 124. Find out how many marbles they had to start with.
10. The height in feet, h, of an object shot upwards into the air with initial velocity, v_o, after t seconds is given by the formula $h = -16t^2 + v_o t$ .A firework is shot upwards with initial velocity $130 \; feet$ per second. How many seconds will it take to reach a height of 260 feet?
11. Factorise the polynomial completely
    a) $f(x) = x^3 + 2x^2$
    b) $f(x) = 3x^3 + x^2 + 3x + 5$
    c) $p(x) = x^3 - 7x^2 + 6x$
    d) $g(x) = (2x^2 - x - 3)(1 + 2x - x^2)$
12. Find the remainder when
    a) $p(x) = 3x^3 + x^2 + 3x + 5 \; is \; divided \; by \; (x + 1)$
    b) $p(x) = x^3 - 3x^2 + 6x - 40 \; divided \; by \;\; (x - 5)$
    c) $p(x) = 6x^3 + 13x^2 - 4 \;\; divided \; by \; (2x + 3)$
13. The polynomial $x^3 + 4x^2 + 7x + k$, where k is a constant is denoted by $f(x)$.
    a) Given that $(x + 2)$ is a factor of $f(x)$,show that $k = 6$
    b) Express $f(x)$ as a product of a linear factor and a quadratic factor.
14. Given $f(x) = 24x^3 + ax^2 - 3x + b$, where a and b are constants. when $f(x)$ is divided by $(2x - 1)$ the remainder is 30
    a) Show that $a + 4b = 114$
    b) Give that $(x + 1)$ is a factor of $f(x)$, find the value of a and b.
    c) Hence factorise the polynomial $f(x)$.
15. Let $f(x) = -6x^3 - 7x^2 + 40x + 21$
    a) Use long division to show that $(x + 3)$ is a factor of $f(x)$

b) Factorise $f(x)$ completely.

c) Hence solve the equation $6(2^{3y}) + 7(2^{2y}) = 40(2^{y}) + 21$

16. Factorise completely and Sketch the following polynomials.

a) $p(x) = x^3 - 4x$

a) $p(x) = 3x^4 + 9x^3 + 6x^2$

b) $f(x) = 3x^3 + x^2 + 3x + +5$